

FACULTY OF SCIENCE
B.Sc. VI-Semester (CBCS) Examination, January 2021

Subject : Mathematics (Vector Calculus)

Paper – VIII-B (DSE) E-2

Time : 2 Hours

Max. Marks: 60

PART – A

Answer any four questions.

(4x5=20 Marks)

- 1 Find the circulation of the vector $\vec{F} = (y, -x, 0)$ around the unit circle $x^2 + y^2 = 1, z = 0$ taken in anticlockwise direction.
- 2 Evaluate the integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (y, x, 0)$ and C is the curve $y = \sin x, z = 0$ between $x = 0$ and $x = \pi$.
- 3 Find the surface integral of $\vec{u} = \vec{r}$ over the surface of the unit cube $0 \leq x, y, z \leq 1$ with n pointing outward.
- 4 Show that any irrotational vector field $\vec{F} = (2x + y, x, 2z)$ is conservative.
- 5 Show that $\int_S \vec{r} \cdot \vec{n} \, ds = 3V$ where V is volume enclosed by the surface S.
- 6 Evaluate $\int_C \frac{ydx - xdy}{x^2 + y^2}$ where C is the unit circle $x^2 + y^2 = 1$.
- 7 Give the geometrical interpretation of gradient.
- 8 For any closed surface S show that $\int_S (\nabla \times \vec{u}) \cdot \vec{n} \, ds = 0$.

PART – B

Answer any two questions.

(2x20=40 Marks)

- 9 Define conservative vector field. Examine whether the vector field $\vec{F} = (y, -x, 0)$ is conservative.
- 10 Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (y^2, x, z)$ C is the curve $y = z = e^x$ from $x = 0$ to $x = 1$
- 11 Define volume integral. Evaluate $\int \int \int \rho \, dV$ where $\rho = 1 + x + y + z$ and $0 \leq x, y, z \leq 1$.
- 12 Find the volume of the tetrahedron with vertices at $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$.
- 13 Find the gradient of the scalar field $f = xyz$ and hence evaluate at the point $(1, 2, 3)$, hence find the directional derivative of f at this point in the direction of the vector $(1, 1, 0)$.
- 14 For what values of a and b the vector field $\vec{u} = (y \cos x + a x z, b \sin x + z, x^2 + y)$ is irrotational.